

Nuclear Saturation and Correlations.

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The relation between nuclear saturation and NN-correlations is examined. Nucleons bound in a nucleus have a reduced effective mass due to the mean field. This results in off-energy-shell scatterings modifying the free-space NN-interaction by a dispersion correction. This is a major contribution to the density-dependence of the effective in-medium force and to saturation. Low-momentum effective interactions have been derived by renormalisation methods whereby correlations may be reduced by effectively cutting off high momentum components of the interaction. The effect of these cut-offs on dispersive corrections and on saturation is the main focus of this paper. The role of the tensor-force, its strength and its effect on correlations is of particular interest. The importance of the definition of the mean field in determining saturation as well as compressibility is also pointed out.

With a cut-off below $\sim 2.6 fm^{-1}$ there is no saturation but at lower density the binding energy is still well approximated suggesting that such a force may be useful in nuclear structure calculations of (small) finite nuclei if saturation is not an issue.

A separable interaction that fits experimental phase-shifts exactly by inverse scattering methods is used. Recent experiments measure short ranged correlations (SRC's) to be 0.23 for ^{56}Fe . Other experiments have obtained a depletion of occupation-numbers in ^{208}Pb to be ~ 0.2 . For nuclear matter with the separable interaction and a continuous spectrum we obtain the related quantity κ to be 0.175 with the Bonn-B deuteron parameters, while Machleidt's gets $\kappa = 0.125$ for the Bonn-B potential and a continuous spectrum.

1 Introduction

The problem of saturation of nuclear forces is nearly as old as nuclear physics but is not yet satisfactorily resolved theoretically. The first efforts to explain the saturation in terms of nuclear forces was the combination of a nonexchange (Wigner) and a space-exchange (Majorana) part. It is found that a Majorana exchange of four times the strength of the Wigner part is required.[1, 2] This disagrees with experimental scattering data. These early studies assumed a "well-behaved" potential such that perturbation theory could be used. The subsequent finding that the 1S_0 phase-shift changes sign at about 250 MeV led Jastrow[3] to propose a very strong short ranged repulsion, a "hard core". This it seemed would also help to explain the saturation property of nuclear matter at least qualitatively. A short ranged repulsion although soft rather than hard has since been part of all modern potentials and is theoretically justified.

The strong nature of the nuclear forces does however exclude a low order perturbation approach as the short-ranged repulsion leads to strong short ranged correlations. The way out of this dilemma was led by Brueckner some 50 years ago developping the theory bearing his name. His approach to solving nuclear many-body problems has dominated all aspects of nuclear physics since, be it nuclear structure or nuclear matter. The problem of nuclear saturation is *qualitatively* solved applying this theory with modern nuclear forces. The saturation is found to result from a combination of the properties of the two-body nucleon force (being repulsive at short distances, being state-dependent and having a tensor-component) and many body effects (correlation effects related to Pauli and dispersion effects). It is generally accepted that the contribution from three-body forces is also important.

Although the problem of the saturation of nuclear forces may be partially resolved with experimentally and theoretically well motivated two- and three- body forces it is still unclear whether the discrepancies found are due to incomplete knowledge of the "free" nuclear forces, the effective interactions in nuclei, nucleonic degrees of freedom or relativistic effects etc. The major topic of the present work is related to our incomplete knowledge of the short-range details of the NN-force as opposed to the long-ranged pion-exchange part. In the "new" approach to the nuclear many-body problem the short-ranged part is 'integrated out' with the goal of reducing the many body problem to that of a low- (or even first-) order problem and eliminating the need of Brueckner or related many-body techniques. In the present work we show the effect on correlations and on saturation as a result of the typical cut-offs in momentum space suggested by this approach.

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We use Brueckner's theory of nuclear matter for our calculations. The central part of this theory is the Reaction matrix K (we use Brueckner's original notation K rather than the often used G to avoid confusion with Green's function), defined by

$$K = V + V \frac{Q}{e} K \quad (1)$$

with V being the "free" NN-interaction, Q/e the in-medium propagator and where Q is the Pauli-operator and e is a sum of kinetic and self-consistent mean field energies

$$U(k) = \sum_{k' < k_F} \langle k k' | K | k k' \rangle \quad (2)$$

The calculations below use this definition of U for both hole and particle states usually referred to as the "continuous" choice. The Brueckner expression for the total energy is given by

$$E = \sum_{k < k_F} \frac{k^2}{2m} + \frac{1}{2} \sum_{k', k < k_F} \langle k k' | K | k k' \rangle \quad (3)$$

The many-body effects of Brueckner's theory are clearly expressed in the Moszkowski-Scott separation method [4, 5]. One particularly important effect as regards saturation is the dispersion correction. K will have off-energy shell contributions due to the momentum dependence of the mean field binding $U(k)$ and this is the origin of this correction.

Let ΔU be the average change in potential energy in intermediate states when solving eq. (1). The dispersion correction to the K -matrix is obtained by differentiating K in eq. (1) by the energy denominator e to get [4]

$$\Delta K_{disp} \propto \Delta U * I_w. \quad (4)$$

where the wound-integral I_w is defined by

$$I_w = \int (\Psi(r) - \Phi(r))^2 d\mathbf{r}$$

with Ψ and Φ the **correlated** and **uncorrelated** two-body wave-functions respectively. The dispersion correction to the energy per particle E/A is then

$$\Delta E_{disp}/A \propto \Delta U * \kappa \quad (5)$$

where $\kappa = \rho * I_w$ with ρ being the density.

The dispersion term is small at low density (small finite nuclei) but grows with density because of the increased binding. It is repulsive and is therefore an important contribution to saturation. It is basically a three-body effect as the effective two-body interaction depends on the mean field due to the presence of "third nucleons" that constitute the mean field U . The wound-integral I_w is an important quantity by itself being a measure of the correlation strengths.

A good understanding of the origin of the dispersion correction is of particular interest with the present "new" approach to the nuclear many-body problem using EFT or similar ideas. In the V_{low-k} approximation with $k < 2 - 3 fm^{-1}$, short-ranged correlations are *a priori* ignored and a nuclear matter calculation gives no saturation, unless supplemented with a three-body interaction of not well-defined origin. In the EFT-approach a low-energy effective interaction is derived but a three-body force is simultaneously generated and may contribute to saturation. This three-body force is however of different origin than the above mentioned dispersion-effect which is a medium or many-body-effect. It is of course now generally accepted that the *bona fide* three-body force is also large enough to be an important factor in understanding the saturation.

The main subject of this paper is to look at the effect of typical momentum cut-offs related to EFT and V_{low-k} and how it relates to two-body correlations in nuclear matter and saturation calculations. In addition we address the more general problem of relation between saturation and nuclear forces.

Section II shows the main results of the numerical work. Part A deals with the dispersion corrections while Part B is concerned with the related depletion factor κ with some results summarized in a Table. It is well-known that tensor correlations in nuclei are important but that they are also to some extent not well determined. We deal with this subject in Section III. In Section IV we discuss some topics related to higher order corrections such as due to the spectral widths and hole-hole propagation. A short Summary and Conclusions are found in Section V.

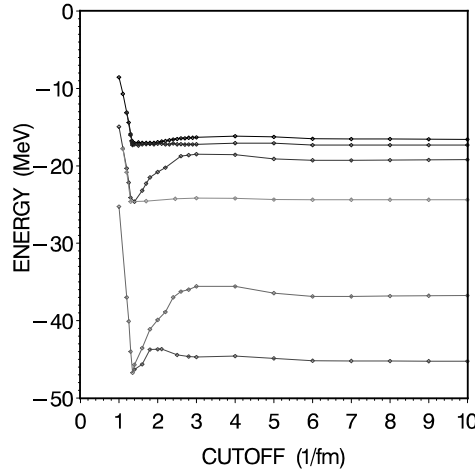


Figure 1: Effects of the selfconsistent mean field (dispersion-correction). There are three sets of curves. The uppermost set shows the contribution to the potential energy per particle from the 1S_0 state, the middle from the 3S_1 and the bottom includes all (21) states. In each set of curves the lower curve is without the mean field $U(k)$ while the upper is with $U(k)$ included in the calculation. The difference between these two curves is the dispersion correction, which is seen to decrease as the cutoff Λ decreases below $\Lambda \sim 3.0 \text{ fm}^{-1}$ and approaches zero as $\Lambda \rightarrow k_F = 1.35 \text{ fm}^{-1}$.

2 Numerical results

The separable potential derived from inverse scattering was calculated as described in previous papers [6, 7]. The Brueckner calculation was done as in numerous previous papers by one of us. The effective interaction $K(\omega, k, P)$ in eq. (1) was calculated as a function of the three variables, the starting energy ω , relative momentum k and center of mass momentum P . The Pauli-operator in eq. (1) was a function of the two variables k and P , the angle-averaged approximation. The sum of the mean fields U in the energy-denominator

$$e = \omega - 2k_i^2 - U(\mathbf{P}/2 + \mathbf{k}_i) - U(\mathbf{P}/2 - \mathbf{k}_i)$$

of eq. (1) was approximated by

$$2 * U((P^2/4 + k_i^2)^{\frac{1}{2}})$$

which implies a quadratic approximation of the mean field around each value of $P/2$. Here k_i are summed over when solving eq. (1). The P contribution to the kinetic energy is cancelled in e . $U(k)$ was calculated from eq.(2) by summing \mathbf{k}' over the fermisea. This involved integrating over the angle between \mathbf{k} and \mathbf{k}' while P and the starting energy ω are functions of this angle. The mean field contribution to the starting energy $\omega = 2k^2 + U(k) + U(k')$ was calculated as above in an effective mass approximation to get $\omega = 2k^2 + 2 * U(\frac{1}{2}(k^2 + k'^2))$. The total energy in eq. (3) was calculated by summing over the mean field $U(k)$.

With present day computing power some of the approximations used above are not necessary. Improved calculations show considerable corrections [8].

2.1 Dispersion effect

The importance of the dispersion effect was emphasized in the Introduction. In this regard we show Fig. 1 which has three sets of graphs, each consisting of two curves.

The upper curve of each set shows the binding energy per particle as a function of cutoff Λ , including dispersion, while the lower is without dispersion. The no dispersion curves are the result of Brueckner calculation

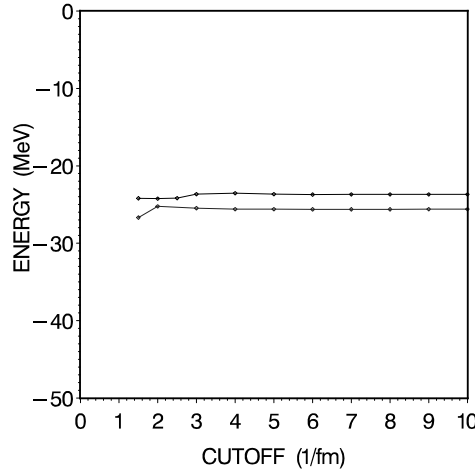


Figure 2: The 3S_1 contribution to the potential energy per particle without the tensor force. The upper curve is with and the lower is without the mean field. It is seen that the difference, the dispersion correction, is small without the tensor force. Compare with the middle set of curves in Fig. 1. The density is here given by $k_F = 1.35 fm^{-1}$

without the selfconsistent mean field U (i.e. with $e \rightarrow e_0$) in eq (1) but with the Q -operator. The uppermost set of curves shows the 1S_0 contribution to the energy per particle while the middle is for the 3S_1 and the lowest set includes all (21) states. We point out that the no-dispersion results are practically independent of cutoff, while the repulsive dispersion results decrease below about $3 fm^{-1}$ to join the no-dispersion result at $\Lambda = k_F$. From the discussion above this serves to show that the effect of the *correlations* between the nucleons is decreasing for cutoffs below $3 fm^{-1}$. The origin of this effect will be discussed further in Section B. This result should also be compared with the $\approx 2 fm^{-1}$ where the phase-shifts turn repulsive. Note also that the dispersion correction for the 1S_0 state is appreciably smaller than for the 3S_1 state. This difference is to be attributed to the tensor correlations. This is further illustrated by Fig. 2 which, when compared with Fig. 1 shows that the short-ranged correlations in the 3S_1 -state contribute relatively little to the total dispersion correction. The dominant correlations are due to the tensor force. Fig. 1 also shows that the dispersions and correlations due to the other states (beyond the S -states) are not negligible.

Fig. 3 shows the importance of the dispersion correction in providing saturation in a Brueckner calculation of the binding energy. The separable interaction without any cut-off is used here. The upper curve is the full Brueckner calculation, while in the lower the selfenergy $U(k)$ is neglected so that the only many-body effect comes from the Q -operator..

The effect of the high momentum cut-off is further illustrated by Fig. 4. With $\Lambda = 9.8 fm^{-1}$ the phase-shifts for all available energies are included in calculating the separable potential while with decreasing Λ these are also decreased in energy accordingly. (See ref [7]). For all the indicated values of Λ the binding energies around and below the experimental saturation density are approximately equal. At higher densities it is however seen that for the smallest value shown here, i.e. $\Lambda = 2.6 fm^{-1}$ there is essentially no saturation. (For $\Lambda = 2 fm^{-1}$ there is of course even less evidence of saturation as shown in Fig. 14.) The effect of the correlations decreases with Λ and the dispersion correction at the higher densities becomes eventually too small to give saturation at a reasonable density. This lack of saturation was also shown in ref [7] and agrees also qualitatively with Fig. 1 of ref [9]. It will be further discussed below in Sect. B in relation to the depletion factor κ .

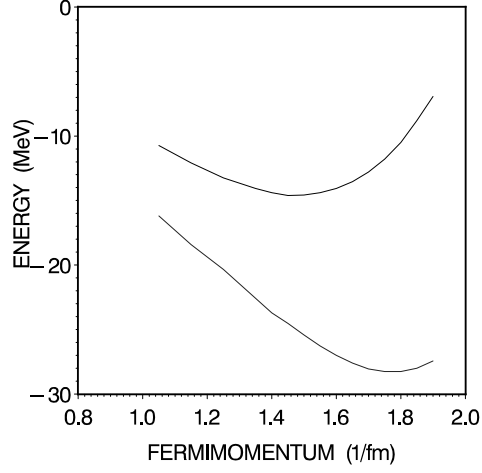


Figure 3: The upper curve is a Brueckner calculation of the total energy per particle as a function of the fermi-momentum. The lower curve shows the result without the selfconsistent mean field U but only kinetic energies (i.e. with $e \rightarrow e_0$) in eq.(1) and therefore no dispersion-correction, while the only many-body effect comes from the Pauli-operator Q .

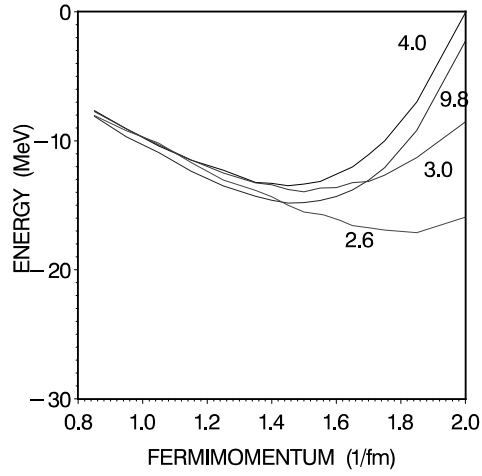


Figure 4: These curves show the effect of high momentum cut-off on the saturation curve. These are Brueckner calculations of the total energy per particle as a function of fermi-momentum for the indicated values of the cut-off momentum Λ in units of fm^{-1} .

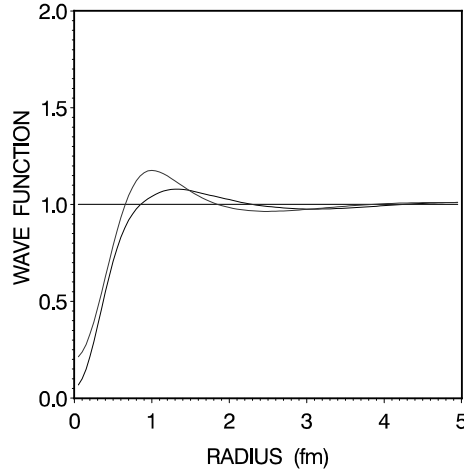


Figure 5: The straight line is the uncorrelated wavefunction Φ at $k = 0$. The lower curve shows the correlated 1S_0 while the upper is the correlated 3S_1 wavefunction Ψ for a relative momentum $k = 0$, a center of mass momentum $P = 0$ and cut-off $\Lambda = 9.8 fm^{-1}$. Note the 'healing'. For the singlet case this gives a $\kappa = .021$ and for the triplet one gets $\kappa = .029$. For small radius $\Psi \rightarrow 0$ and this is evidence of a short-ranged repulsion.

2.2 Depletion factor κ

The (short-ranged) correlations cause scatterings to states outside the fermisea. This results in the depletion of the normally occupied (model) states.[10, 11, 12] This is quantified by κ that we define as (see e.g. refs. [13, 14])

$$\kappa_i = \frac{1}{8}(2J+1)(2T+1)\rho * I_w \quad (6)$$

where the wound-integral I_w was defined above and ρ is the density. κ is state-dependent as is I_w and the explicit calculation requires knowing the correlated (or defect) wave-function for each state-label i . To obtain the reaction matrix K , the inverse scattering method used here does not require an explicit calculation of the correlated wave-functions. With K known they are however obtained from the relation

$$\Psi = \Phi + \frac{Q}{e}K. \quad (7)$$

Fig. 5 shows the results for the singlet and triplet interactions respectively at normal nuclear matter density. Note the relatively larger defect in the triplet case for $r \sim 1 fm$. It is due to the tensor-force. For the triplet case we also show the S-D defect wavefunction in Fig. 6. For $k = 0$ the unperturbed functions $\Phi = 1$ are shown by the straight lines and the well-known healing is evident. Fig. 7 shows the dependence on relative momentum when compared with the 3S_1 case in Fig. 5. Note especially the difference at radius $r = 0$ which is an indication of the momentum dependence (non-locality) of our separable interaction weakening the short-ranged repulsion with increasing momentum. The Figs 5 to 7 are with cutoff $\Lambda = 9.8$. Our results with $\Lambda = 2.0$ are shown in Fig. 8.

One finds in Fig. 8 that the short ranged repulsive effects are completely absent in the correlated wave-functions for the cutoff $\Lambda = 2.0$.

The correlated wavefunctions and (more relevant) the defect wavefunctions and κ 's depend on the center of mass and relative momenta. In the Table below we show averaged values of κ calculated from [4, 12]

$$\kappa_i = \frac{1}{2} \frac{\partial P_i}{\partial U} \quad (8)$$

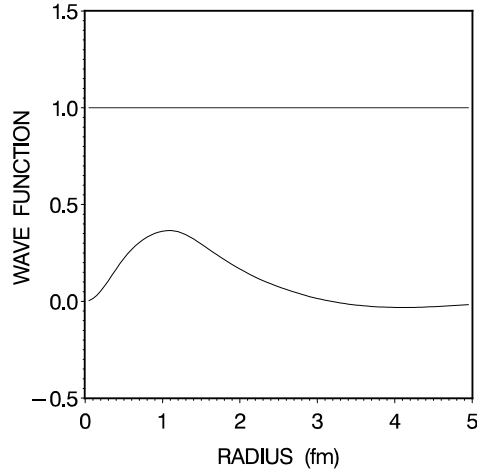


Figure 6: The SD defect wavefunction is shown for a relative momentum $k = 0$ and center of mass momentum $P = 0$. The corresponding $\kappa = .21$.

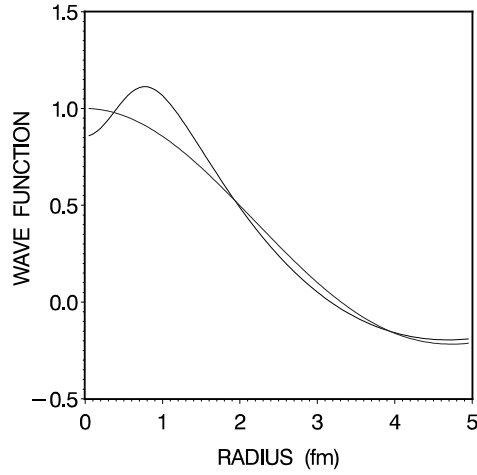


Figure 7: The correlated and uncorrelated 3S_1 wavefunctions for a relative momentum $k = 0.95 fm^{-1}$, center of mass momentum $P = 0$ and cut-off $\Lambda = 9.8$. Comparison with the triplet case for $k = 0$ in Fig. 5 shows that the wavefunction at $r = 0$ is much larger, evidently because of the momentum dependence of our two-body potential. Note the 'healing'. Here $\kappa_{ss} = 0.05$ i.e. larger than for $k = 0$.

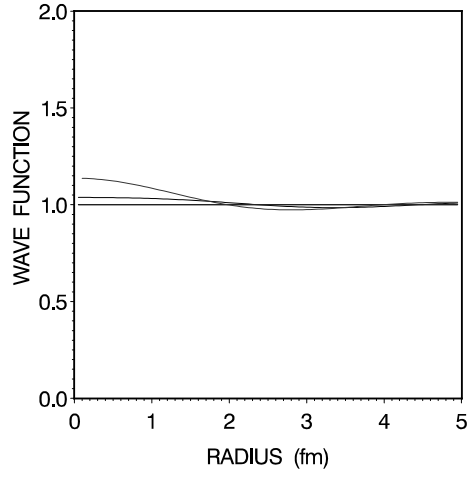


Figure 8: The straight line is the uncorrelated wavefunction Φ at $k = 0$. The lower curve shows the correlated 1S_0 and the upper the correlated 3S_1 wavefunction Ψ for a relative momentum $k = 0$, center of mass momentum $P = 0$ and $\Lambda = 2.0$. Compare with the singlet case in Fig. 5 for $\Lambda = 9.8$. Here $\kappa = .015$ for the 1S_0 less than the value for $\Lambda = 9.8$ consistent with the independently calculated average value of κ for shown in the Table below. Compare also with the triplet case in Fig. 5 for $\Lambda = 9.8$. In this case $\kappa_{ss} = .013$ There is no evidence of a short-ranged repulsion for this value of $\Lambda = 2.0$.

where P_i is the potential energy for state i . The derivative is performed numerically by differentiating the potential energy P_i with respect to increments $\pm 0.1 \text{ MeV}$ in the selfconsistent hole states $U(k)$ for $k < k_F$.

TABLE of κ 's

	$k_F = 1.35$					$k_F = 1.7$	
State	$\Lambda = 9.8$	$\Lambda = 4.0$	$\Lambda = 2.6$	$\Lambda = 2.0$	$\Lambda = 1.6$	$\Lambda = 9.8$	$\Lambda = 2.6$
3S_1	0.109	0.103	0.080	0.062	0.041	0.085	0.045
1S_0	0.016	0.016	0.013	0.012	0.011	0.012	0.006
3P_2	0.009	0.009	0.007	0.005	0.003	0.011	0.006
1P_1	0.006	0.006	0.005	0.004	0.002	0.008	0.005
3D_2	0.005	0.005	0.004	0.004	0.002	0.003	0.004
3P_1	0.024	0.021	0.014	0.009	0.004	0.039	0.015
1D_2	0.002	0.002	0.001	0.001	0.001	0.003	0.001
3P_0	0.002	0.002	0.001	0.001	0.001	0.002	0.001
Total	0.175	0.163	0.124	0.095	0.061	0.172	0.088

The Table shows that the averaged κ 's decrease with the cutoff $\Lambda < 4.0 \text{ fm}^{-1}$ consistent with the Figs 5 and 8 where the correlated wave functions show a very large Λ dependence especially at *small radii*. The argument that κ is closely associated with the saturation property is substantiated by Fig. 4 that shows the nuclear matter saturation to decrease for $\Lambda < 4 \text{ fm}^{-1}$ and this is consistent with Fig. 1 that shows the dispersion effect to decrease with $\Lambda < 2.6 \text{ fm}^{-1}$.

The Table above also shows κ 's for twice nuclear matter density ($k_F = 1.7 \text{ fm}^{-1}$). It is especially noticable that the κ for the 3S_1 state and $\Lambda = 9.8$ has decreased by $\sim 20\%$ relative to its value at normal nuclear matter density. For $\Lambda = 2.6$ it has decreased by 50% . This has to be attributed to the larger effect of the Pauli-blocking cutting off the low-momentum component of the SD excitations. To substantiate this we show Fig. 11 to be compared with the 3S_1 curve in Fig. 5. The correlations around $r = 1 \text{ fm}$ have decreased by about 10% . It is however also seen from the Table that for $\Lambda = 9.8 \text{ fm}^{-1}$ the total κ at $k_F = 1.7 \text{ fm}^{-1}$ is practically unchanged from its value at $k_F = 1.35$, but for $\Lambda = 2.6$ it has decreased by $\sim 40\%$. We remind that κ is in fact proportional to both ρ and to I_w so that unless I_w decreases with density as seems to be the case for the 3S_1 state one does indeed expect κ to increase with density as is also seen for some but not all of the other states. It may be of interest to note that calculations with the Hamada-Johnston potential showed a substantial increase of κ with density [11] increasing from $\sim .24$ at $k_F = 1.4$ to $\sim .40$ at $k_F = 1.8$ a reflection of the fact that the short-ranged structure of this potential is quite different. (Table 10 in ref [11] shows I_w as a function of k and k_F). It can also be pointed out that in the Separation Method[4] the wound-integral is calculated from the "free" short-ranged correlations and therefore independent of density.

We like to emphasize that the density *dependence* of κ (and the dispersion correction) is of importance in determining not only the saturation density but also the compressibility. Likewise we like to point out that the dispersion correction to the energy (eq. (5)) is proportional not only to κ but also to the excitation ΔU .

If Λ decreases we have in fact the following scenario. The short-ranged correlations decrease. As a consequence the wound-integral and κ decreases. Another consequence is that the excitations to higher energies where the mean field is less attractive or even repulsive are suppressed AND the available phase-space is cut off at the higher end to contribute to this suppression. At the lower end it is the Q -operator that cuts off. One therefore finds that both κ and ΔU decrease with Λ as a consequence of the decreased correlations. So the result is that the dispersion ΔE_{disp} in eq. 5 decreases not only because of a decrease in κ but also because of a smaller ΔU . In fact, if Λ is decreased to k_F there are no excitations as they are completely suppressed by the Pauli-operator and Λ and the dispersion correction will be zero. This is clearly seen in Fig. 1.

Figs 9 and 10 further illustrate the situation. Both Figures are practically identical for momenta $k < 2 \text{ fm}^{-1}$. From the Table it is seen that the κ for the 1S_0 state is only slightly smaller for $\Lambda = 2.0$ than for $\Lambda = 9.8$ but the dispersion corrections are quite different because of the different cutoffs.

An additional factor to consider in this discussion is that the correlations in the medium i.e. the κ 's also depend on the chosen mean field that of course is the origin of the off-shell scatterings. This is well-known and exemplified by the different κ 's obtained with the "continuous" and "standard" choices of single particle energies. (See e.g. ref [15]). One concludes that any discussion of the saturation has to involve the definition of the mean field. It does not only involve the correlations.

Higher order correlation effects are discussed in Sect. IV.

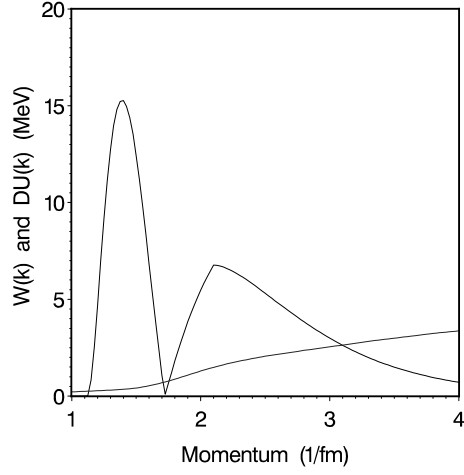


Figure 9: The defect wavefunction $W(k) = (\Psi(k) - \Phi(k))^2$ is shown together with $DU(k) = U(k'_1) + U(k'_2) - U(k_1) - U(k_2)$, the latter in units of $\hbar^2/2m$. The convolution of these functions gives a dispersion-correction of 1.7MeV . The contribution to this dispersion from momenta $k < 2\text{fm}^{-1}$ is 0.08MeV . Compare with Fig. 10 for $\Lambda = 2$. Here $\Lambda = 9.8\text{fm}^{-1}$ and $P = 1.5\text{fm}^{-1}$, $k = 0.45\text{fm}^{-1}$.

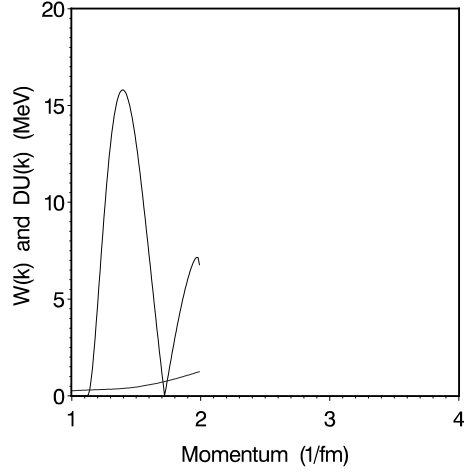


Figure 10: Similar to Fig. 9 except that $\Lambda = 2.0\text{fm}^{-1}$. The dispersion correction is here obtained to be 0.10MeV nearly the same as the contribution to dispersion for $k < 2$ in Fig. 9.

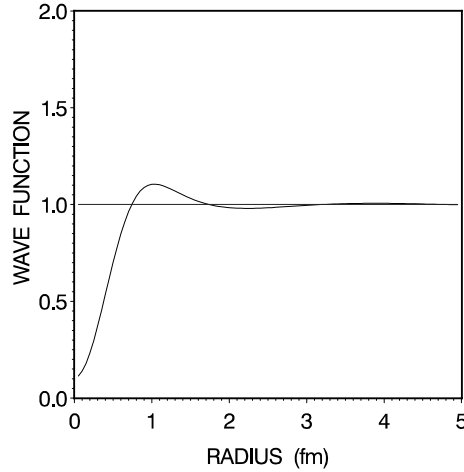


Figure 11: The correlated and uncorrelated 3S_1 wavefunctions for a relative momentum $k = 0$, center of mass momentum $P = 0$ and cut-off $\Lambda = 9.8 fm^{-1}$ at twice normal nuclear matter density. Comparison with the 3S_1 curve in Fig. 5 shows a slight decrease in correlations at this higher density. This would explain the decrease in $\kappa_{{}^3S_1}$ with increased density.

It is noted that our results for κ 's are slightly larger than those reported by Machleidt [15]. He reports a total $\kappa = 0.125$ for the Bonn-B potential with a continuous spectrum. Our larger value of $\kappa = 0.175$ is consistent with the larger (more effective) saturation obtained with our potential[6]. The methods for calculating the two κ 's are however not the same. Machleidt (presumably following ref [13]) calculates κ from the wound-integral by eq (6) at some averaged center of mass and relative momenta, while our averaged κ 's are calculated from eq (8).

The κ does depend on the strength of the tensor-force. We find for comparison a $\kappa = .196$ when using the Bonn-C deuteron wavefunctions which gives a $P_D = 5.6$ and $\kappa = .162$ with the Bonn-A having $P_D = 4.4$ while Bonn-B has a $P_D = 5.0$ with $\kappa = .175$.

The κ is a measure of the correlations in nuclei and the probability of nucleons being scattered out of the fermi-sea. This depletion can also be calculated from the spectral function $S(p, \omega)$ with

$$\rho(p) = \frac{1}{2\pi} \int S(p, \omega) d\omega,$$

with $\kappa = 1 - \rho(p)$. This has not been done here but a result of such a calculation is shown in Fig. 12 taken from ref. [35]. It is rather typical of several such calculations found in the literature that give an occupation at or near 0.8 agreeing with the measurements quoted below. It is only slightly smaller at the fermi surface. At twice the density the Figure shows however a significant decrease in occupation i.e. an increase of κ . This is contrary to the result shown in the Table above. This increase in κ is however not as large as quoted above for the Hamada-Johnston potential that gave $\kappa = .4$ at this density. It was already pointed out above that the density-dependence of κ is important not only in determining saturation but also compressibility and it also has consequences for astrophysical theories. Measurements of spectroscopic factors in ${}^{208}Pb$ finds depletions of $0.22 \pm 0.02 \pm 0.06$ for deep-lying states [34], that within error-bars agrees with our result.

An apparently related quantity (the per-nucleon probability for two nucleon Short Ranged Correlations) is measured in recent experiments [36] to be 0.15, 0.19, and 0.23 for 4He , ${}^{12}C$, and ${}^{56}Fe$ respectively. These values are larger than the κ 's shown in our calculations above. This would require even stronger correlations. The exact interpretation of the experiments may still have to be clarified however. The κ for the 3S_1 state is seen above to be ~ 6 times as large as the κ for the 1S_0 state. This is consistent with recent experiments [37].

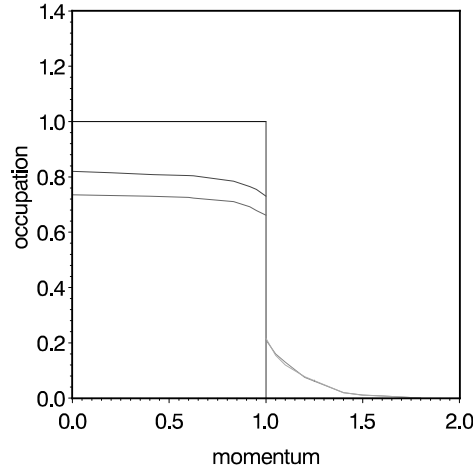


Figure 12: Occupation numbers $\rho(p)$ at normal (upper curve) and twice (lower curve) normal nuclear matter density. The momentum is in units of k/k_F and $k_F = 1.35 \text{ fm}^{-1}$. The uncorrelated $T = 0$ distribution ("square curve") is also shown for reference.

3 Tensor correlations

The numerical results above agree with the well-known fact that a dominant contribution to the dispersion-term and saturation comes from the tensor-component.(see e.g. ref [15]) The short ranged correlations in the 1S_0 state and in the 3S_1 state with the tensor force switched off contribute much less to the dispersion correction as shown by the numerical results in Figs 1 and 2. This situation leads however to a big dilemma because of the experimental and theoretical uncertainty regarding the strength of the tensor-force at short distances, which is an important factor in calculating the D state probability P_D in the deuteron. The long-ranged part is of course accurately determined by the pion-exchange contribution. While the deuteron quadrupole-moment Q and the asymptotic ratio $\eta = D/S$ are well known experimentally the D -state probability P_D is only known approximately, being somewhere between 4 and 7 %. This dilemma was already emphasized by Machleidt who defined three phase-shift equivalent potentials Bonn-A, Bonn-B and Bonn-C having different tensor-strengths with values of P_D being 4.4, 5.0 and 5.6 respectively.[15] These potentials gave very different saturation properties. This was also shown to be the case with the inverse scattering potentials used in the present work.[6] For a fixed range of the deuteron wavefunctions the saturation energy and density will decrease with the D -state probability P_D . [15, 6] The relation between saturation and the uncertainty of the deuteron wavefunction was further illustrated by the class of potentials named FBS in ref [6]. These were derived from deuteron wavefunctions constrained by the deuteron-data (η and Q) but with longer tails in momentum space. Parametrical fits to the $d(e, e'p)n$ experimental data by Bernheim *et al*[16] available for $k < 1.7 \text{ fm}^{-1}$ were made. Increasing the range of the wavefunctions i.e assuming a longer tail in momentum space (in the region not determined experimentally), but keeping P_D , Q and η fixed, will have an effect similar to that of decreasing P_D . [6] The P_D is however mainly determined by the wave-function in the region around $k = 2 \text{ fm}^{-1}$ not quite available from the Bernheim *et al* data that were limited to $p < 335 \text{ MeV}/c$. The need to go to higher energies to fully explore the D -state distribution was also stressed by Bernheim *et al*.

More recent efforts to explicitly determine the tensor-component in the NN-interaction were made by measuring elastic electron-deuteron scatterings for momentum transfers up to $Q^2 = 1.7 (\text{GeV}/c^2)$. [17] Of particular interest here are the T_{20} data which should be closely related to the tensor-force. Comparisons of these scattering data with relativistic calculations[18] as well as QCD[19, 20] show the former to agree fairly well with the data while not the latter. It is however well recognised that the meson theoretical calculations are hampered

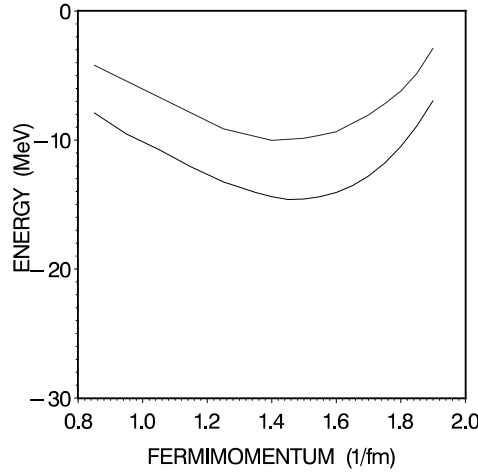


Figure 13: The lower curve is the result of a Brueckner calculation (i.e. with particle-particle ladders only) of the total energy as a function of density (Fermimomentum). The lower curve includes hole-hole ladders perturbatively by changing the Pauli-operator in eq. 1.

by incomplete knowledge of meson exchange currents (MEC).

The question of the strength of the tensor-force and therefore its role in the problem of nuclear saturation is therefore still unresolved at this time. It may in fact be the most important unsolved problem in nuclear (many body) physics.

4 Higher Order Corrections

The results presented here were all obtained in the Brueckner approximation given by eqs. (1,2,3). Higher order rearrangement corrections were already discussed and estimated by Brueckner and Goldman [21]. and in many subsequent works e.g. refs. ([12, 22, 23, 26, 25]). More recent but related work has concentrated on corrections due to spectral selfconsistency and inclusion of hole-hole ladders.

The Brueckner K -matrix includes only particle ladders while hole-hole ladders are also naturally included in Green's function techniques. It was shown that, in the quasi-classical limit, the Green's function mean field $Re\Sigma^+ = U + ReU^{(2)}$ with $U^{(2)}$ the Brueckner second order rearrangement[12]. In this limit hole-hole ladders can therefore be included simply by including $U^{(2)}$ in the calculation of the total energy E from eq. (3). Our result of such a calculation is shown in Fig. 13. This is a perturbative calculation. Higher order is well known to be divergent. The lower curve is a Brueckner calculation while the upper includes the hole-hole ladders. It is seen to decrease the binding by $\sim 4MeV$.

The spectral broadening was first calculated from the second order Brueckner rearrangement energy [22]. It was then also concluded that the width stems from the long-ranged part of the interaction. In a subsequent work this broadening of the spectral function was included selfconsistently in a nuclear matter calculation [30]. This was done using Green's function techniques. Only the long-ranged part of an interaction was used so that any effect of shortranged correlations was not included. It was found that the spectral broadening increased the binding energy and also slightly the saturation density. While the long-ranged correlations give a spectral broadening but essentially no depletion the shortranged correlations do cause depletion.

In ref [23] the effect of this depletion on the selfenergy was calculated from the Brueckner third order rearrangement energy. This is a short-ranged effect and it results in a "renormalisation" of the mean field to

$$U(k) \rightarrow (1 - \kappa U(k)).$$

When reinserted in eq. (1) the result is an increased binding as a result of decreased dispersion-correction by eq. (4). The saturation density is also increased in such a calculation. The Brueckner total energy given by eq. (3) is however a quasi-particle approximation summing over occupation numbers 0 and 1. It was shown in ref. [26] that this can overestimate the binding relative to a summation over spectroscopic strengths as in Koltun's sum rule.

The above results indicate and it has been shown repeatedly that, if one goes beyond the quasi-particle approximation, spectral selfconsistency should be invoked. The *ab initio* quasi-particle (Brueckner) calculations have been extended by several authors utilizing Green's function formalisms thereby including corrections due to the broadening and depletion of the spectral functions as well as the hole-hole ladders. With present-day computer-power this is now feasible [9, 29, 27, 28]. Consequently the width- and depletion-effects calculated above are now included selfconsistently as they are imbedded in the spectral functions. Comparison with Brueckner calculations shows a decrease in binding very similar to what is seen in Fig. 13[29]. This can be regarded as a relatively small correction and shows near cancellation of the different effects mentioned above.

This extension of Brueckner's original theory does however not address other higher order effects that may affect two-body correlations and saturation. One issue is for example alternative insertions in particle lines that would affect the ΔU in eq. (4) for the dispersion correction. A rigorous treatment of these insertions was made by Bethe who realised that they should be treated as 3-body collisions [32]. It has however been claimed by Song *et al* [33] that all three-hole line contributions are essentially included if the continuous choice of the spectrum is used as we have also done here.

We believe that there are still uncertainties relating to both the NN-interaction at short distances including the tensor component and in the neglect of higher order corrections in the many-body theory. Another issue is of course 3-body forces that we have not included in this work.

The present evaluation of corrections discussed above does however not alter our main conclusions regarding saturation and the cut-off parameter Λ although it can not be ruled out that our values of κ can be slightly changed by improved computing techniques [8]. This should be investigated. It was however shown that the Brueckner result in eq. 8 agrees with the Green's function result if using the EQP- (Extended Quasi-Particle-) approximation for the spectral function. (See eq. (66) in ref. [12]).

5 Summary and Conclusions

Many-body calculations with realistic nuclear forces have a long history. Various methods have been used: Brueckner, e^S , coupled cluster, HNC and maybe others. These more or less agree in that realistic 2-body forces and a plausible 3-body force can provide reasonable saturation of nuclear matter and experimental fits to the lightest nuclei. The effective "in-medium" 2-body force is density-dependent and the 3-body force can also be regarded as an effective density-dependent 2-body force. Without any reference to a specific theory one can therefore conclude that the effective force in the nuclear medium has to be density-dependent in order to achieve the observed saturation. The origin of this density-dependence is however only partially understood theoretically.

The philosophy behind the V_{low-k} and EFT methods is that the relative momenta of nucleons in nuclei are low and that the high-momentum components of the NN-interaction therefore should be allowed to be integrated out since they are not well known anyway. The result is a renormalised "smooth" effective force that may even be treated in low order perturbation theory.

The V_{low-k} effective force does in itself not provide saturation and has to be supplemented with a $3N$ force of not well defined origin.[31] The effective force generated by EFT-methods does explicitly generate a 3-body force but apparently not sufficiently strong to provide saturation.

In Brueckner theory the effective force is obtained by replacing the free space propagator $1/e_0$ in the nuclear medium by Q/e . The Q operator as well as the mean field included in the energy-denominator e contribute to the density-dependence. The momentum-dependent (non-local) mean field results in off-shell scatterings in the many-body medium which together with short-ranged correlations yields a dispersion term (eq. (4)) which is density-dependent. This is a major contributor to the density-dependence of the effective force in Brueckner theory. The range of the correlations contributing to this density-dependence is typically long (in momentum space) compared to the fermi-momentum and to the typical cut-offs used to produce the renormalised low-momentum effective forces. The question addressed in this paper is which effect the cut-off has on the density-dependence and consequently the saturation and the compressibility of nuclear matter.

The study is done with a separable potential derived by previously published inverse scattering methods. [6] The input consists of the scattering phase-shifts and the deuteron data. The effect of the cut-off of the high momentum phase-shifts on the effective interaction was shown in a previous publication [7] and the similarity with V_{low-k} was shown. In the present work a primary interest has been the dispersion effects.

The effect of the cut-offs on the dispersion correction was shown in Figs 1-2. It is found that at normal saturation density a cut-off larger than about $3fm^{-1}$ is necessary to include the full effect of the correlations on dispersion and the saturation property of the force.

We reaffirm the importance of tensor-correlations in Section III. They are not fully defined by the scattering phase-shifts but the deuteron properties are important inputs. It was already shown by Machleidt [15] that deuteron wave-functions constrained by the known deuteron quadrupole moment and the asymptotic D/S ratio but having different unknown D-state probabilities give different saturation results.[15] This was confirmed by the inverse scattering results of Kwong and Köhler [6], who further investigated the effect on saturation using three additional model wave-functions for the deuteron. The D-state admixture is largely unknown for momenta above $\sim 2fm^{-1}$. This leaves a major uncertainty in the theory of saturation.

It is now generally accepted that an important part of the density-dependence comes from 3-body forces. It seems however that at least part of the unknown factor regarding saturation rests with two-body correlations especially in the 3S_1 channel. Because of this uncertainty it seems reasonable to look for alternatives. One such is the Moscow-potential of Neudatchin *et al* [38, 39] that has a strong short-ranged attraction that changes the phase-shifts by an unobservable amount of π radians resulting in a node in the relative wavefunction. This leads to a strongly correlated short-ranged system resulting in larger wound-integrals and dispersion corrections. If the tensor force is not sufficient to give sufficient saturation this force may be the right answer to the saturation problem.

We have here only been concerned with the effect of correlations and momentum cut-offs in nuclear matter calculations. A somewhat different problem is that of the effect of cut-offs for nuclear structure calculations. It is however shown in Fig. 4 that for densities below the saturation density, the total binding energy is practically independent of cut-off at least for $\Lambda > 2.6fm^{-1}$. This suggests that short-ranged correlation effects may be of less importance in finite nuclei as long as saturation is not an issue, i.e. if the density distribution is constrained by fixing the nuclear size. If not, the calculation would result in a finite nucleus collapsing to a too small radius.

One motivation to develop low momentum effective interactions has been that it might be used in low order perturbation theory as opposed to the more traditional many-body techniques developed for strongly interacting media. [31] In Fig. 14 are shown four different curves. These are results of two Brueckner, one first and one second order calculation of nuclear matter binding energies, the latter with a cut-off $\Lambda = 2.0$. As expected there is no sign of saturation for $\Lambda = 2.0$. This was already seen to be the case for the Brueckner calculation with $\Lambda = 2.6fm^{-1}$ in Fig. 4. However, the second order result shows a considerable improvement over the first order at densities below saturation approaching the Brueckner result with the same Λ .

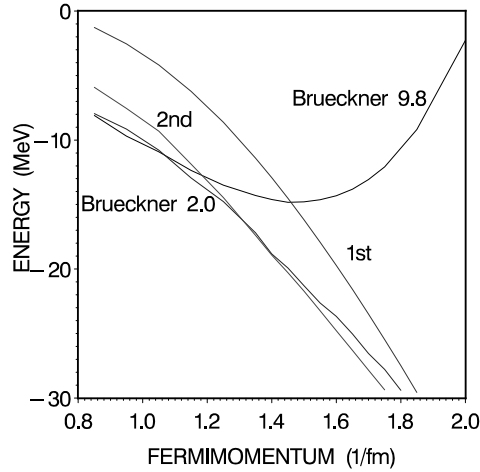


Figure 14: Brueckner calculations of the binding energy per particle for $\Lambda = 9.8$ and 2.0 fm^{-1} together with first and second order calculations with an interaction defined with a cut-off $\Lambda = 2.0$.

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